# FEniCS Course

## Lecture 5: Happy hacking
## Tools, tips and coding practices

*Contributors*
André Massing

fenics project

# Post-processing

# Function evaluation

`Expression` and `Function` objects `f` can be evaluated at arbitrary points:

```
# 1D
x = 0.5
f(x)
# 2D
x = (0.5,0.3) # tuple
# x = [0.5,0.3] is also valid
f(x)
# 3D
x = (0.5,0.2,1.0) # tuple
# x = [0.5,0.2,1.0] is also valid
f(x)
print f(x)
```

Short-hand

```
    f(0.5,0.5)
```

Exercise: Try it out! Use one of your existing codes and evaluate the solution at some point.

# Function evalution vs. Function representation

Question: What about plotting $\sin(u_h)$? And $\nabla u_h$ and $|\nabla u_h|$?
Experiment: Try it out! Use

```
sqrt(inner(grad(u),grad(u)))
```

for $|\nabla u|$. What happens if you plot these function? Have a closer look at the terminal output. Anything suspicious?

Question: What happened now? Why is there a
`> Object cannot be plotted directly, projecting to piecewise linears.`
Answer:

- $\sin(u_h(x))$ is the evaluation of the built-in function sin at a *given* value $u_h(x)$, which in turn results from a FEM function evalution.
- $\sin \circ u_h$ is a composition of the built-in function sin and a FEM function $u_h$. The composition is a `UFL` (Unified Form Language) expression.

# Simple code validation

# Theory can help you to validate your implementation!

## *A priori* estimates for the Poisson problem

If

- $u \in H_0^1(\Omega) \cap H^{k+1}(\Omega)$
- $V_h = \{v_h \in C(\Omega) : v_h \in P^k(T) \; \forall \, T \in \mathcal{T}\}$

then

$$E_1(h) := \|u - u_h\|_{1,\Omega} \leq C h^k \|u\|_{k+1,\Omega}$$
$$E_0(h) := \|u - u_h\|_{0,\Omega} \leq C h^{k+1} \|u\|_{k+1,\Omega}$$

where $\|\cdot\|_{l,\Omega} = \|\cdot\|_{H^l(\Omega)}$ for $l = 0, 1, k+1$.

# Theory can help you to validate your implementation!

## *A priori* estimates for the Poisson problem

If

- $u \in H_0^1(\Omega) \cap H^{k+1}(\Omega)$
- $V_h = \{v_h \in C(\Omega) : v_h \in P^k(T) \; \forall \, T \in \mathcal{T}\}$

then

$$E_1(h) := \|u - u_h\|_{1,\Omega} \leq Ch^k \|u\|_{k+1,\Omega}$$
$$E_0(h) := \|u - u_h\|_{0,\Omega} \leq Ch^{k+1} \|u\|_{k+1,\Omega}$$

where $\| \cdot \|_{l,\Omega} = \| \cdot \|_{H^l(\Omega)}$ for $l = 0, 1, k+1$.
Taking log on each side

$$\log(E_1(h)) \leq \log(Ch^k \|u\|_{k+1,\Omega}) = k \log(h) + \log(C\|u\|_{k+1,\Omega})$$

# Theory can help you to validate your implementation!

## *A priori* estimates for the Poisson problem

If

- $u \in H_0^1(\Omega) \cap H^{k+1}(\Omega)$
- $V_h = \{v_h \in C(\Omega) : v_h \in P^k(T)\ \forall\, T \in \mathcal{T}\}$

then

$$E_1(h) := \|u - u_h\|_{1,\Omega} \leq C h^k \|u\|_{k+1,\Omega}$$
$$E_0(h) := \|u - u_h\|_{0,\Omega} \leq C h^{k+1} \|u\|_{k+1,\Omega}$$

where $\|\cdot\|_{l,\Omega} = \|\cdot\|_{H^l(\Omega)}$ for $l = 0, 1, k+1$.
Take the log of each side:

$$\underbrace{\log(E_1(h))}_{y} \leq \log(C h^k \|u\|_{k+1,\Omega}) = k \underbrace{\log(h)}_{x} + \underbrace{\log(C\|u\|_{k+1,\Omega})}_{c}$$

# Method of manufactured solutions

## Recipe

1. Take a suitable function $u$
2. Compute $-\Delta u$ to obtain $f$
3. Compute boundary values (trivial if only Dirichlet boundary conditions are used)
4. Solve the corresponding variational problem

$$a(u_h, v) = L(v)$$

   for a sequence of meshes $\mathcal{T}_h$ and compute the error $E_i(h) = \|u - u_h\|_{i,\Omega_i}$ for $i = 0, 1$
5. Plot $\log(E_i(h))$ against $\log(h)$ and determine $k$

## Homework

Try this by taking $u = \sin(2\pi x)\sin(2\pi y)$ on the unit square. Solve the problem for $N = 2, 4, 8, 16, 64, 128$ and compute both the $L^2$ and $H^1$ errors for $P1$, $P2$ and $P3$ elements as a function of $h$. Can you determine the convergence rate?