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Physics as Flow of Information as Computation as FenICs as G2

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FEniCS 06

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FenICs: Automated G2 Computation

- Physics: Analog Computation: Fields
- Information Flow: Analog/Digital Comp
- Computation: Digital (ev. Fields)
- Physics as Automated Computation
- as Turing Machine
- as Automatable (Constructable) Mathematics
- PRECISION of Physics Computation??
- STABILITY of Physics Computation??

Physics

- Classical Mechanics: Particles
- Solid Mechanics: Elastic Bodies: Fields
- Fluid Mechanics: Continuum: Fields
- Thermodynamics–Statistical Mech: Particles
- Quantum Mechanics: Atoms: Fields
- Relativity Theory: Galaxies
- QM + Relativity =???
- Action at Distance???
- Fields Do Exist, Particles Not

Automation vs Observation/Control

- Automated:
 - No Observation, No Control
 - No-Mind, No Intelligence
- Non-Automated:
 - Observation, Control
 - Mind(s), Intelligence

Flow of Information

- Collect: Local: Code
- Communicate: Waves/Flow: Global
- Receive: Local: Decode
- Communication
- Propagation: Node to Neighbor Node
- Iterated Local: Global
- Shannon Information Theory: Flow Entropy
- PRECISION–STABILITY–Capacity

Iterative Poisson Solver

$$\Delta\varphi = \rho$$

$$\varphi(x) = \Delta^{-1}\rho(x) = \int \frac{\rho(y)dy}{4\pi|x-y|} \quad \text{Global}$$

$$\epsilon\dot{\varphi} - \Delta\varphi = -\rho \quad \text{Local Relaxation}$$

- Jacobi-Iteration
- Local Communication with Neighbors
- No Assembly: No El. Stiffness Matrix: Action
- Global: Fiction
- (Iterated) Local: Reality

One-Mind: Super-Intelligence

- Full Information Centralized Global
- Infinite Speed of Communication
- Observation-Control
- Parallel Computing: SIMD
- ONE SUPER-PHYSICIST, BIG BROTHER
- Totalitarian
- Fixed Step Jacobi Relaxation
- Does not seem to work well??

Many-Minds: Limited Intelligence

- Partial Information Decentralized
- Finite Speed of Communication
- Partial Observation-Control
- Parallel Computing: MIMD
- MANY small physicists: Democratic
- Explicit Time-Stepping
- Variable Step Jacobi Relaxation
- Seems to work reasonably well??

Democratic System

- Individual Decisions by Many
- General Principles: Law
- Parallell Computation: MIMD
- Minimal Information for Local Computation
- Nobody Knows Everything
- Parallel Limited Intelligence
- Adaptive by Feed-Back
- Variable Relaxation Jacobi

Totalitarian System

- BIG BROTHER: Decides 5 Year Plan
- Individuals Follow 5 Year Plan
- BB: Knows Everything
- BB: Understands Everything
- Individual: No-Mind
- Non-Adaptive No Feed-Back
- Fixed Relaxation Jacobi
- BB Gaussian Elimination: Assembly

Classical Mech: Newton: Gravitation

- Particles: Mass M_j , Pos. $u_j(t)$, $j=1, \dots, N$,
- Conservation of Momentum $m_j = M_j \dot{u}_j$

$$\dot{m}_j = \nabla \sum_{k \neq j} \frac{GM_j M_k}{|u_j - u_k|}$$

- G Gravitational Constant
- Grav. Force: Action at Distance: Non-Local
- How?? Gravitons??
- Explicit Time-Stepping

Fluids: Cons of Mass Mom Energy

Find $\hat{u} \equiv (\rho, m, e)$ with $m = \rho u$ such that

$$\begin{aligned}\dot{\rho} + \nabla \cdot (\rho u) &= 0, \\ \dot{m} + \nabla \cdot (mu + p\delta) &= 0, \\ \dot{e} + \nabla \cdot (eu + pu) &= 0, \\ \hat{u}(\cdot, 0) &= \hat{u}^0,\end{aligned}$$

$$p = (\gamma - 1)\rho T, \quad e = \rho|u|^2/2 + \rho T,$$

- Local Balance: Finite Speed of Propag.
- Explicit Time-Stepping

Cosmology: Grav. Non-Local

Find $\hat{u} \equiv (\rho, m, e, \varphi)$ such that

$$\begin{aligned}\dot{\rho} + \nabla \cdot (\rho u) &= 0, \\ \dot{m} + \nabla \cdot (mu + (p - G\varphi)\delta) &= 0, \\ \dot{e} + \nabla \cdot (eu + pu) &= 0, \\ \Delta\varphi &= \rho, \\ \hat{u}(\cdot, 0) &= \hat{u}^0,\end{aligned}$$

- $\rho \rightarrow \varphi = \Delta^{-1}\rho$ Non-Local
- Mass ρ creates Gravitational Pot ϕ : How??
- Explicit Time-Stepping.

New Cosmology: Mass Creation Local

Eliminate ρ by $\rho = \Delta\varphi$: Find $\hat{u} = (\varphi, \chi, m, e)$:

$$\begin{aligned}\dot{\varphi} &= \chi, \\ \epsilon\dot{\chi} - \Delta\chi - \nabla \cdot m &= 0, \\ \dot{m} + \nabla \cdot (mu + (p - G\varphi)\delta) &= 0, \\ \dot{e} + \nabla \cdot (eu + (p - G\varphi)u) &= 0, \\ \hat{u}(\cdot, 0) &= \hat{u}^0,\end{aligned}$$

- ϵ relaxation parameter, $u = \frac{m}{\Delta\varphi}$
- $\varphi \rightarrow \rho = \Delta\varphi$: Grav Pot Creates Mass: How??
- Local, Time-Stepping

New Cosmology: Creationism

- Potential Peak:

$$\varphi(x) \approx \Phi = -\frac{1}{|x|} \rightarrow \Delta\Phi = \delta \quad \text{Unit Mass}$$

- Dark Matter: Force but no Visible Mass

- Mollified Peak: Force but no Visible Mass??

- Create Pot Peaks: $\varphi = \pm\Phi$ from $\varphi = 0$

- Matter-Antimatter: Repulsion (Neg Grav)

Quantum Mechanics: Schrödinger Eq

Wave function $\psi(t, x_1, \dots, x_N)$ for N electrons:

$$i\dot{\psi} + \left(\sum_j \Delta_j + \sum_j \frac{N}{|x_j|} - \sum_{k \neq j} \frac{1}{2|x_j - x_k|} \right) \psi = 0$$

- Walter Kohn Nobel Prize 1998:
- ψ Not “Legitimate Scientific Object”
- ψ does NOT EXIST for $N \geq 100$
- Non-Local, Time-Stepping, SpaceDim: $= 3N$

QM: Hartree Approx: Non-Local

For $j = 1, \dots, N$, find $\psi_j(t, x)$:

$$i\dot{\psi}_j + \left(\Delta + \frac{N}{|x|} - W_j\right)\psi_j = 0$$

$$W_j(x, t) = \sum_{k \neq j} \int \frac{|\psi_k(y, t)|^2}{2|x_j - y|} dy \approx \sum_{k \neq j} \frac{1}{2|x_j - \bar{x}_k|}$$

$$\psi(t, x_1, \dots, x_N) \approx \psi_1(t, x_1) \dots \psi_N(t, x_N)$$

■ Non-Local Time-Stepping

■ SpaceDim: = 3 Systemdim: N

Many-Minds Quantum Mechanics

$$i\dot{\psi}_j + \left(\Delta + \frac{N}{|x|} - W_j\right)\psi_j = 0$$

$$\epsilon\dot{W}_j - \Delta W_j = -\frac{1}{2} \sum_{k \neq j} |\psi_k|^2$$

- Electron j (One-Mind) Solves Its Equation
- Influence from $k \neq j$ into W_j
- No Pauli Exclusion Princ (Antisym ψ)
- PEP Vaild?? Simply Coulomb Repulsion??

Special Relativity

- Speed of Light = $c = 1$ Independent of
- Speed of Source/Observer
- Lorenz Transformation: Inertial Systems
- Invariance of Wave/Maxwell
- Difficult to Understand:
- Paradoxes: Time Dilation, Twins....
- Impossible to Verify Experimentally
- Cannot be Wrong (Pseudo-Scientific Popper)

Many-Minds Relativity

Minds $1, \dots, N$: Minds j and k Agree on

- Mutual Distance $|u_j - u_k|$
- Unit of Length: Lightsecond
- Speed of Light = $c = 1$ lightsecond/second
- But Not on $|u_j - u_i|$ for $i \neq k \dots$
- Minimal Info for Gravitational Interaction
- No BIG BROTHER

Relativistic Addition of Velocities

$$0 < v_1, v_2 < 1$$

$$\bar{v} = v_1 + v_2 - v_1 v_2.$$

$$0 < \bar{v} < 1$$

Relativistic Newton's Law

Velocity/Acc vs Fixed Observer in Approach:

$$\frac{m}{1 - v} \dot{v} = F$$

Velocity/Acc vs Fixed Observer Moving Away:

$$\frac{m}{1 + v} \dot{v} = F$$

Acceleration vs Co-Moving Observer (Itself):

$$m \dot{v} = F$$

Many-Minds QM-Relativity

$$i\dot{\psi}_j + \left(\Delta + \frac{N}{|x|} - W_j + G\varphi_j\right)\psi_j = 0$$

$$\epsilon\dot{W}_j - \Delta W_j = -\frac{1}{2} \sum_{k \neq j} |\psi_k|^2$$

$$\epsilon\dot{\varphi}_j - \Delta\varphi_j = -\sum_{k \neq j} M_j M_k |\psi_k|^2$$

- Electron j : Co-Moving Coord Syst
- Relativistic Corr of Mass if Fixed System

Fluid Model

- Variable Density Incompressible Fluid
- Cylinder: $0 \leq x_1 \leq L$, $|\bar{x}| \leq R$
- $\Gamma^- = \{x_1 = 0\}$ inflow, $\Gamma^+ = \{x_1 = L\}$ outflow

$$\begin{aligned}\dot{\rho} + \nabla \cdot (\rho u) &= 0, \\ \dot{u} + u \cdot \nabla u + \frac{\nabla p}{\rho} &= 0, \\ \nabla \cdot u &= 0, \\ u \cdot n &= u^- \quad \rho = \rho^- \quad \text{on } \Gamma^-, \\ \rho &= \rho^+ \quad \text{on } \Gamma^+, \end{aligned}$$

ρ^- input movie on Γ^- , ρ^+ output movie on Γ^+

Collection-Transmission-Reception

- ρ^- input movie,
- Transport by Flow
- ρ^+ output movie.
- Turbulent Flow
- Distortion/Error: $\rho^- - \rho^+$
- Point Values vs Mean Values

Physics as Comp: Thermodynamics

- Class 2nd Law: $TdS = dT + pdV$, $dS \geq 0$,
- S Entropy: $dS = dT/T - (\gamma - 1)d\rho/\rho$ Exact!!
- $S = \log(T/\rho^{\gamma-1}) = \log(p/\rho^\gamma)$ State Variable
- Class Question: What is S ? Nobody Knows.
- New 2nd Law: $dT + pdV \geq 0$, No S . Forget It.
- New 2nd Law: G2: $dT + pdV = (hR, R) \geq 0$
- Local Destruction of Information
- Not Trivial Vanishing Viscosity

Physics as Comp: Paradox Resolution

- Loschmidt: Irreversibility in Hamiltonian Syst
- Gibbs: Inextensivity of S
- PRECISION vs STABILITY (No Statistics)
- Interpret Computation as Physics
- Understand Physics as Computation
- Is the World a G2 World??
- Galerkin: Res. Weakly Small: Share Local
- LS Stab: Res. Pointw. Too Not Big

G2 LS Stabilization

- Penalty for Breaking Law
- Positive Penalty
- 2nd Law
- Destruction of Information

Physics: Analog Computation

- Massive
- Parallel
- Local
- PRECISION
- STABILITY
- DESTRUCTION of INFORMATION
- NO STATISTICS!!

FenICs/G2 as Physics as VISION

- Turbulence Body&Soul Vol 4, Jan 07
- Thermodynamics Vol 5, 07
- MM Quantum Mechanics Vol 6, 08??
- MM Relativity Vol 7, 08??
- Advanced Applications!!
- Generality of FenICs/G2
- Simplicity of FenICs/G2
- Efficiency of FenICs/G2