Functions over Meshes

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Outline

1. Review of Mesh
2. Section Interface
3. Completion
4. Conclusions

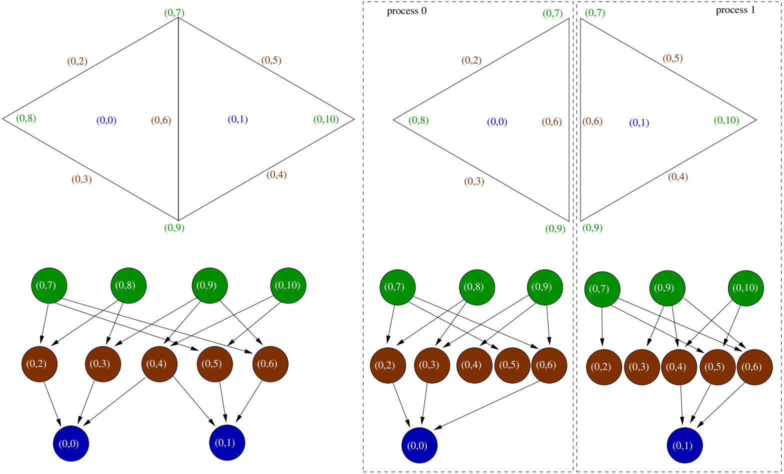
Part I

Review of Mesh

A *Topology* is a collection of points with a covering relation

- Points represent vertices, edge, . . .
- Covering relation is represented by directed edges
  - This produces a graph, called a *Sieve*
  - For meshes, the graph is DAG and is stratified
- In a `Topology`, we allow multiple `Sieves`

Sieve: Distributed Mesh



Bundle

A *Bundle* is an association of spaces to points

A *Section* is a function over these spaces

- A `Bundle` combines `Sections` with a `Topology`
- A `Mesh` is a `Bundle` over the computational topology
  - It has a distinguished Section, *coordinates*
  - The intrinsic dimension is the height/depth of the Sieve

Part II

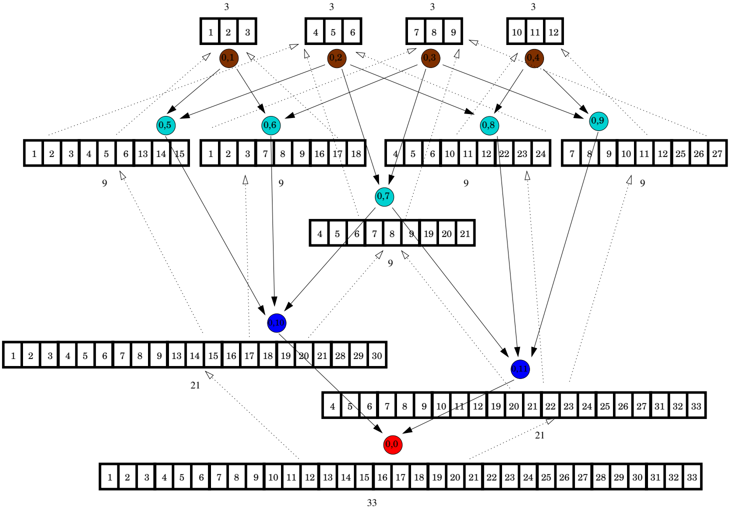
Section Interface

Section Interface

A `Section` is a mapping from Sieve points to a vector of values

- `restrict`, `update`
  - Defined on the `closure` of a point
- Use an *Atlas* to manage dimension of each fiber
  - Can be implemented by a `Section`
  - Must also manage the domain (harder)
- Participate in *completion*
  - Communicate values over an *Overlap*

Sieve: Mesh Data



Part III

Completion

Section Completion

An *Overlap* is a Sieve associating points in different Sieves

We have four phases:

- Copy local values
- Communicate sizes
  - Notice that the size is constant
- Communicate values
- Update section with remote values

Communication

We use auxiliary objects having a Section interface.

- Use a *sizer* to allocate overlap section
  - Use an Atlas and `restrict` to the point
  - Use section interface for overlap section
  - Just the completion of the Atlas
- Use a *filler* to update overlap section
  - Use a Section and `restrict` to the point
- Communicate values in overlap sections
  - Can use an arbitrary fusion strategy, not just addition or replacement

General communication routines can enable

- Mesh partition/distribution (and unification)
- Section distribution
- Load balancing

Mesh Completion

Meshes can be reduced to sections

- Discrete topology is a section over the partitions
  - Complete this section to distribute points
- Topology is a section over the discrete topology
  - Values are *cones*, in the space of points
  - Complete this section to distribute cones
- Complete associated sections

Section Implementations

- `ConstantSection`
  - Constant value over the domain
  - No communication to complete
- `UniformSection`
  - Constant fiber dimension
  - Atlas can be a `ConstantSection`
- `Section`
  - Arbitrary fiber dimension
  - Atlas can be a `UniformSection`
- Thus we have termination of a completion recursion

Concepts vs. Types

Concepts

- Sieve
- Overlap
- Section
- Atlas
- Bundle

Types

- `Sifter`
- `Sifter`
- `ConstantSection`
- `UniformSection`
- `Mesh`

Custom Assembly

To define a given assembly, we need

- Domain definition
- Overlap Construction
- Fusion operator

This could support

- FETI-DP, BDDC
- GMG
- FMM

Part IV

Conclusions

Conclusions

- Must distinguish between Concept and Type
    - Soon to be included in C++
- Can make do with two basic objects
    - Sieve
    - Section
- This vastly simplifies algorithms
    - Most notable in communication

Part V

Ideas about Build Systems

Traditional Problems

- Global namespace
  - SCons continues this make shortcoming
- Configuration and build dependencies
  - No explicit hierarchy or dependencies
- Audit trail for configure/build information
  - When did this flag/library which broke my test get included?
- Integration of configure and build
  - Uniform, structured access to configure data
- Configuration of batch systems
- Persistence

Proposals

- Encapsulation
  - Configure data in Python objects
  - Use framework `require` to access configure objects
  - Pass in build object to make rules
- Auditing
  - Some kind of transition log for designated variables
- Configure integration
  - Simple `require()` interface to the configure DAG
- Configure extensibility
  - Configure object template
- Configure for batch systems
  - Generate and build a C executable, which runs in the queue
  - This generates `reconfigure.py` which sets options correctly
- Persistence
  - Use builtin Python persistence

Dependencies

- Configure uses the framework `require('module.name', self)`
  - Returns the requested configure object
  - Creates a DAG edge between that object and `self`
  - Could be extended to interproject dependencies?
- Build still using text BNF-style
  - Should establish a full DAG underneath (broken in recursive make)
  - Auto-dependencies?

Configure Integration

- `addDefine()`, `addSubstitution()`
    - Replicates the Autoconf interface
- `addTypedef()`, `addPrototype()`
    - Better interaction with C/C++
- `addMakeMacro()`, `addMakeRule()`
    - Structured interface to `make`
- Custom build rules
    - Determine includes, libraries, and flags directly from configure
    - Can establish implicit rules
    - Use automake-like targets

Build Rules

```
def bin_foo(maker):
  'foo: foo.c bar.h'
  return
```

```
def dylib_foo(maker):
  'libfoo: foo.c bar.c'
  return (maker.mpi.include, maker.mpi.lib, [ '-DFOO'])
```

```
def dylib_bat(maker):
  'libbat: bat.c'
  return ([], [os.path.join(maker.libDir,  'libfoo.a')], [
```

Other Features

Sometimes incidental features can greatly increase usability

- PETSc Options and configure argument parsing
- Configure help system
- Integrated version control
- `importer.py`