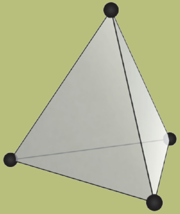
2 **P-**



4

P$_1$ $\mathcal{P}_1^- \Lambda^0(\Delta_3)$

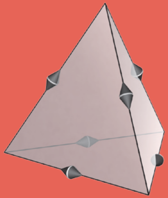
$4 \times \underbrace{\mathcal{P}_0 \Lambda^0(\Delta_0)}_{1} = 4$

```
("P", tetrahedron, 1)
```

2 P-

3 P-

6



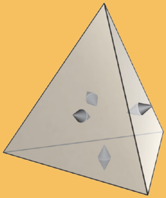
N1$_1^e$ $\mathcal{P}_1^-\Lambda^1(\Delta_3)$

$6 \times \underbrace{\mathcal{P}_0\Lambda^0(\Delta_1)}_{1} = 6$

```
("N1E", tetrahedron, 1)
```

3 P-

4 *P-*



4

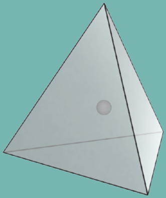
N1$^{\mathbf{f}}_{\mathbf{1}}$ $\mathcal{P}_1^- \Lambda^2(\Delta_3)$

$4 \times \underbrace{\mathcal{P}_0\Lambda^0(\Delta_2)}_{1} = 4$

```
("N1F", tetrahedron, 1)
```

4 *P-*

5 ***P-***



1

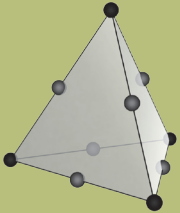
dP_0 $\mathcal{P}_1^-\Lambda^3(\Delta_3)$

$1 \times \underbrace{\mathcal{P}_0\Lambda^0(\Delta_3)}_{1} = 1$

```
("DP", tetrahedron, 0)
```

5 ***P-***

6 P-



10

$\mathbf{P_2}$

$\mathcal{P}_2^- \Lambda^0(\Delta_3)$

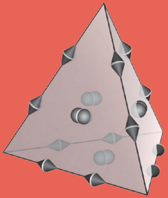
$4 \times \underbrace{\mathcal{P}_1\Lambda^0(\Delta_0)}_{1} + 6 \times \underbrace{\mathcal{P}_0\Lambda^1(\Delta_1)}_{1} = 10$

`("P", tetrahedron, 2)`

P- 9

7 P-

20



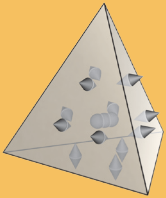
$\mathbf{N1^{e}_{2}}$ $\mathcal{P}_2^- \Lambda^1(\Delta_3)$

$$6 \times \underbrace{\mathcal{P}_1\Lambda^0(\Delta_1)}_{2} + 4 \times \underbrace{\mathcal{P}_0\Lambda^1(\Delta_2)}_{2} = 20$$

```
("N1E", tetrahedron, 2)
```

7 P-

8 P-



15

N1$^f_2$ $\mathcal{P}_2^- \Lambda^2(\Delta_3)$

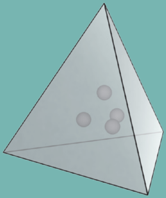
$$4 \times \underbrace{\mathcal{P}_1\Lambda^0(\Delta_2)}_{3} + 1 \times \underbrace{\mathcal{P}_0\Lambda^1(\Delta_3)}_{3} = 15$$

`("N1F", tetrahedron, 2)`

-d 8

9 **P-**

4



$\mathbf{dP_1}$

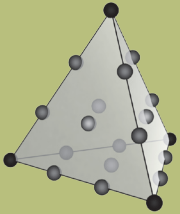
$\mathcal{P}_2^- \Lambda^3(\Delta_3)$

$1 \times \underbrace{\mathcal{P}_1 \Lambda^0(\Delta_3)}_{4} = 4$

```
("DP", tetrahedron, 1)
```

9 **P-**

10 P-



20

P$_3$

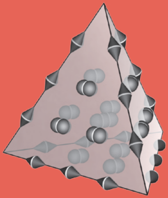
$\mathcal{P}_3^- \Lambda^0(\Delta_3)$

$$4 \times \underbrace{\mathcal{P}_2\Lambda^0(\Delta_0)}_{1} + 6 \times \underbrace{\mathcal{P}_1\Lambda^1(\Delta_1)}_{2} + 4 \times \underbrace{\mathcal{P}_0\Lambda^2(\Delta_2)}_{1} = 20$$

```
("P", tetrahedron, 3)
```

10 P-

$\int$ **P-**



45

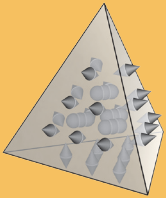
N1$^{e}_{3}$ $\mathcal{P}_3^- \Lambda^1(\Delta_3)$

$$6 \times \underbrace{\mathcal{P}_2\Lambda^0(\Delta_1)}_{3} + 4 \times \underbrace{\mathcal{P}_1\Lambda^1(\Delta_2)}_{6} + 1 \times \underbrace{\mathcal{P}_0\Lambda^2(\Delta_3)}_{3} = 45$$

`("N1E", tetrahedron, 3)`

$\int$ **P-**

Q **P-**



36

N1$^{\mathbf{f}}_{\mathbf{3}}$

$\mathcal{P}_3^- \Lambda^2(\Delta_3)$

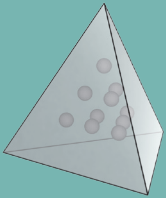
$4 \times \underbrace{\mathcal{P}_2\Lambda^0(\Delta_2)}_{6} + 1 \times \underbrace{\mathcal{P}_1\Lambda^1(\Delta_3)}_{12} = 36$

```
("N1F", tetrahedron, 3)
```

P- Q

K P-

10



dP_2

$\mathcal{P}_3^- \Lambda^3(\Delta_3)$

$1 \times \underbrace{\mathcal{P}_2\Lambda^0(\Delta_3)}_{10} = 10$

```
("DP", tetrahedron, 2)
```

K P-

A P-

$\mathcal{P}_r^- \Lambda^k$

The shape function space for $\mathcal{P}_r^- \Lambda^k$ is

$$\mathcal{P}_{r-1}\Lambda^k + \kappa \mathcal{P}_{r-1}\Lambda^{k+1},$$

where κ is the Koszul differential.[7] It includes the full polynomial space $\mathcal{P}_{r-1}\Lambda^k$, is included in $\mathcal{P}_r\Lambda^k$, and has dimension

$$\dim \mathcal{P}_r^- \Lambda^k(\Delta_n) = \binom{r+n}{r+k}\binom{r+k-1}{k}.$$

The degrees of freedom are given on faces f of dimension $d \geq k$ by moments of the trace weighted by a full polynomial space:

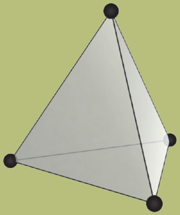
$$u \mapsto \int_f (\mathrm{tr}_f\, u) \wedge q, \quad q \in \mathcal{P}_{r+k-d-1}\Lambda^{d-k}(f).$$

The spaces with constant degree r form a complex:

$$\mathcal{P}_r^- \Lambda^0 \xrightarrow{\mathrm{d}} \mathcal{P}_r^- \Lambda^1 \xrightarrow{\mathrm{d}} \cdots \xrightarrow{\mathrm{d}} \mathcal{P}_r^- \Lambda^n.$$

A P-

2 P



4

P_1

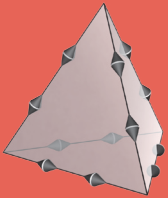
$\mathcal{P}_1\Lambda^0(\Delta_3)$

$4 \times \underbrace{\mathcal{P}_1^-\Lambda^0(\Delta_0)}_{1} = 4$

("P", tetrahedron, 1)

2 P

3 *P*



12

N2$^{e}_{1}$

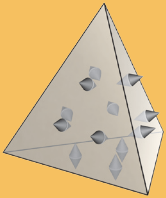
$\mathcal{P}_1\Lambda^1(\Delta_3)$

$6 \times \underbrace{\mathcal{P}_1^-\Lambda^0(\Delta_1)}_{2} = 12$

("N2E", tetrahedron, 1)

3 *P*

4 P



12

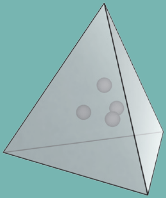
N2$^{f}_{1}$ $\mathcal{P}_1\Lambda^2(\Delta_3)$

$4 \times \underbrace{\mathcal{P}_1^-\Lambda^0(\Delta_2)}_{3} = 12$

("N2F", tetrahedron, 1)

4 P

5 P



4

dP_1

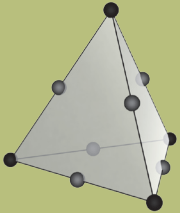
$\mathcal{P}_1\Lambda^3(\Delta_3)$

$1 \times \underbrace{\mathcal{P}_1^-\Lambda^0(\Delta_3)}_{4} = 4$

```
("DP", tetrahedron, 1)
```

5 P

6 P



10

$\mathbf{P_2}$

$\mathcal{P}_2\Lambda^0(\Delta_3)$

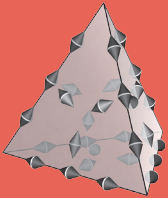
$$4 \times \underbrace{\mathcal{P}_2^- \Lambda^0(\Delta_0)}_{1} + 6 \times \underbrace{\mathcal{P}_1^- \Lambda^1(\Delta_1)}_{1} = 10$$

```
("P", tetrahedron, 2)
```

6 P

7 P

30



N2$^{e}_{2}$ $\mathcal{P}_2\Lambda^1(\Delta_3)$

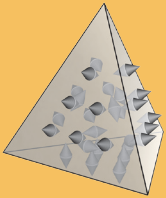
$6 \times \underbrace{\mathcal{P}_2^-\Lambda^0(\Delta_1)}_{3} + 4 \times \underbrace{\mathcal{P}_1^-\Lambda^1(\Delta_2)}_{3} = 30$

("N2E", tetrahedron, 2)

7 P

8 *P*

30



N2$^{f}_{2}$

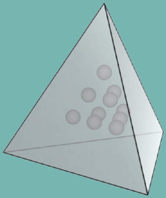
$\mathcal{P}_2\Lambda^2(\Delta_3)$

$4 \times \underbrace{\mathcal{P}_2^-\Lambda^0(\Delta_2)}_{6} + 1 \times \underbrace{\mathcal{P}_1^-\Lambda^1(\Delta_3)}_{6} = 30$

("N2F", tetrahedron, 2)

d 8

9 P



10

dP_2

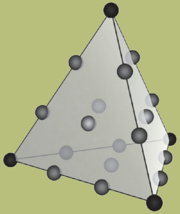
$\mathcal{P}_2\Lambda^3(\Delta_3)$

$1 \times \underbrace{\mathcal{P}_2^-\Lambda^0(\Delta_3)}_{10} = 10$

("DP", tetrahedron, 2)

d 6

10 ***P***



20

P_3

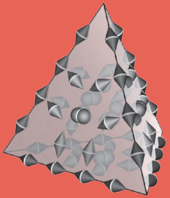
$\mathcal{P}_3\Lambda^0(\Delta_3)$

$$4 \times \underbrace{\mathcal{P}_3^-\Lambda^0(\Delta_0)}_{1} + 6 \times \underbrace{\mathcal{P}_2^-\Lambda^1(\Delta_1)}_{2} + 4 \times \underbrace{\mathcal{P}_1^-\Lambda^2(\Delta_2)}_{1} = 20$$

("P", tetrahedron, 3)

10 P

J P



60

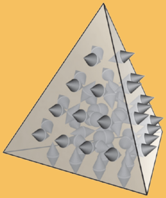
N2$^{e}_{3}$ $\mathcal{P}_3\Lambda^1(\Delta_3)$

$$6 \times \underbrace{\mathcal{P}_3^-\Lambda^0(\Delta_1)}_{4} + 4 \times \underbrace{\mathcal{P}_2^-\Lambda^1(\Delta_2)}_{8} + 1 \times \underbrace{\mathcal{P}_1^-\Lambda^2(\Delta_3)}_{4} = 60$$

```
("N2E", tetrahedron, 3)
```

J P

Q *P*



60

$\mathbf{N2}^{\mathbf{f}}_{\mathbf{3}}$ $\mathcal{P}_3\Lambda^2(\Delta_3)$

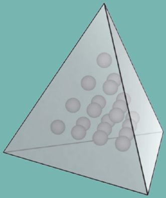
$$4 \times \underbrace{\mathcal{P}_3^-\Lambda^0(\Delta_2)}_{10} + 1 \times \underbrace{\mathcal{P}_2^-\Lambda^1(\Delta_3)}_{20} = 60$$

```
("N2F", tetrahedron, 3)
```

Q *P*

K **P**

20



dP$_3$ $\mathcal{P}_3\Lambda^3(\Delta_3)$

$1 \times \underbrace{\mathcal{P}_3^-\Lambda^0(\Delta_3)}_{20} = 20$

("DP", tetrahedron, 3)

K **P**

A P

$\mathcal{P}_r\Lambda^k$

The shape function space for $\mathcal{P}_r\Lambda^k$ consists of all differential k-forms with polynomial coefficients of degree at most r, and has dimension

$$\dim \mathcal{P}_r\Lambda^k(\Delta_n) = \binom{r+n}{r+k}\binom{r+k}{k}.$$

The degrees of freedom are given on faces f of dimension $d \geq k$ by moments of the trace weighted by a $\mathcal{P}_r^-$ space:

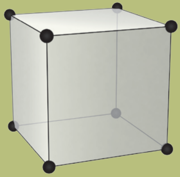
$$u \mapsto \int_f (\mathrm{tr}_f\, u) \wedge q, \quad q \in \mathcal{P}^-_{r+k-d}\Lambda^{d-k}(f).$$

The spaces with decreasing degree r form a complex:

$$\mathcal{P}_r\Lambda^0 \xrightarrow{\mathrm{d}} \mathcal{P}_{r-1}\Lambda^1 \xrightarrow{\mathrm{d}} \cdots \xrightarrow{\mathrm{d}} \mathcal{P}_{r-n}\Lambda^n.$$

A P

2 Q-



8

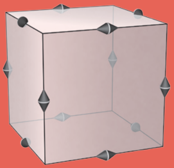
Q_1 $\mathcal{Q}_1^- \Lambda^0(\square_3)$

$8 \times \underbrace{\mathcal{Q}_0^- \Lambda^0(\square_0)}_{1} = 8$

("Q", hexahedron, 1)

2 Q-

3 Q-



12

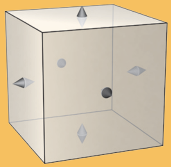
Nc_1^e $\mathcal{Q}_1^-\Lambda^1(\square_3)$

$12 \times \underbrace{\mathcal{Q}_0^-\Lambda^0(\square_1)}_{1} = 12$

```
("NCE", hexahedron, 1)
```

3 Q-

4 Q-



6

$\mathbf{Nc}_1^{\mathbf{f}}$

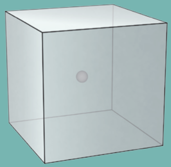
$\mathcal{Q}_1^- \Lambda^2(\square_3)$

$6 \times \underbrace{\mathcal{Q}_0^- \Lambda^0(\square_2)}_{1} = 6$

```
("NCF", hexahedron, 1)
```

4 Q-

5 Q-



1

d$\mathbf{Q_0}$

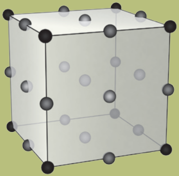
$\mathcal{Q}_1^- \Lambda^3(\square_3)$

$$1 \times \underbrace{\mathcal{Q}_0^- \Lambda^0(\square_3)}_{1} = 1$$

```
("DQ", hexahedron, 0)
```

5 Q-

6 Q-



27

Q_2

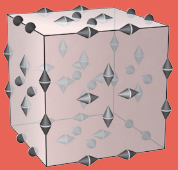
$\mathcal{Q}_2^- \Lambda^0(\square_3)$

$$8 \times \underbrace{\mathcal{Q}_1^- \Lambda^0(\square_0)}_{1} + 12 \times \underbrace{\mathcal{Q}_1^- \Lambda^1(\square_1)}_{1} + 6 \times \underbrace{\mathcal{Q}_1^- \Lambda^2(\square_2)}_{1} + 1 \times \underbrace{\mathcal{Q}_1^- \Lambda^3(\square_3)}_{1} = 27$$

`("Q", hexahedron, 2)`

6 Q-

7 $\mathcal{Q}^-$



54

$\mathbf{Nc_2^e}$

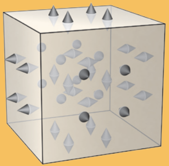
$\mathcal{Q}_2^-\Lambda^1(\square_3)$

$$12 \times \underbrace{\mathcal{Q}_1^-\Lambda^0(\square_1)}_{2} + 6 \times \underbrace{\mathcal{Q}_1^-\Lambda^1(\square_2)}_{4} + 1 \times \underbrace{\mathcal{Q}_1^-\Lambda^2(\square_3)}_{6} = 54$$

```
("NCE", hexahedron, 2)
```

7 $\mathcal{Q}^-$

8 Q-



36

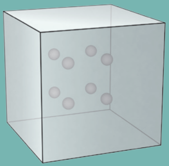
$\mathbf{Nc_2^f}$ $\mathcal{Q}_2^- \Lambda^2(\square_3)$

$6 \times \underbrace{\mathcal{Q}_1^- \Lambda^0(\square_2)}_{4} + 1 \times \underbrace{\mathcal{Q}_1^- \Lambda^1(\square_3)}_{12} = 36$

("NCF", hexahedron, 2)

-Q 8

9 Q-



8

dQ$_1$

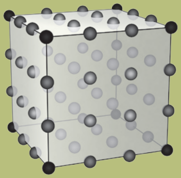
$\mathcal{Q}_2^- \Lambda^3(\square_3)$

$1 \times \underbrace{\mathcal{Q}_1^- \Lambda^0(\square_3)}_{8} = 8$

`("DQ", hexahedron, 1)`

9 Q-

10 Q-



64

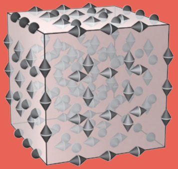
Q_3

$\mathcal{Q}_3^- \Lambda^0(\square_3)$

$8 \times \underbrace{\mathcal{Q}_2^- \Lambda^0(\square_0)}_{1} + 12 \times \underbrace{\mathcal{Q}_2^- \Lambda^1(\square_1)}_{2} + 6 \times \underbrace{\mathcal{Q}_2^- \Lambda^2(\square_2)}_{4} + 1 \times \underbrace{\mathcal{Q}_2^- \Lambda^3(\square_3)}_{8} = 64$

```
("Q", hexahedron, 3)
```

10 Q-



144

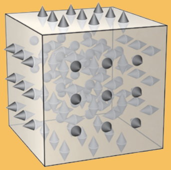
$\mathbf{Nc}_3^{\mathbf{e}}$

$\mathcal{Q}_3^- \Lambda^1(\square_3)$

$$12 \times \underbrace{\mathcal{Q}_2^- \Lambda^0(\square_1)}_{3} + 6 \times \underbrace{\mathcal{Q}_2^- \Lambda^1(\square_2)}_{12} + 1 \times \underbrace{\mathcal{Q}_2^- \Lambda^2(\square_3)}_{36} = 144$$

```
("NCE", hexahedron, 3)
```

Q Q-



108

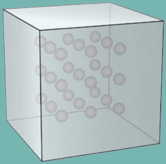
$\mathbf{Nc}_3^f$ $\mathcal{Q}_3^- \Lambda^2(\square_3)$

$$6 \times \underbrace{\mathcal{Q}_2^- \Lambda^0(\square_2)}_{9} + 1 \times \underbrace{\mathcal{Q}_2^- \Lambda^1(\square_3)}_{54} = 108$$

`("NCF", hexahedron, 3)`

Q Q-

K Q-



27

dQ_2 $\mathcal{Q}_3^- \Lambda^3(\square_3)$

$$1 \times \underbrace{\mathcal{Q}_2^- \Lambda^0(\square_3)}_{27} = 27$$

```
("DQ", hexahedron, 2)
```

A Q-

$\mathcal{Q}_r^- \Lambda^k$

This family is constructed from the complex of 1-dimensional finite elements using a tensor product construction.[10] The shape function space on the unit cube $\square_n = I^n$ is given by

$$\bigoplus_{\sigma \in \Sigma(k,n)} \left[\bigotimes_{i=1}^{n} \mathcal{P}_{r-\delta_{i,\sigma}}(I) \right] \mathrm{d}x^{\sigma_1} \wedge \cdots \wedge \mathrm{d}x^{\sigma_k},$$

where $\Sigma(k,n)$ denotes the increasing maps $\{1, \ldots, k\} \to \{1, \ldots, n\}$. Its dimension is $\dim \mathcal{Q}_r^- \Lambda^k(\square_n) = \binom{n}{k} r^k (r+1)^{n-k}$. The degrees of freedom are

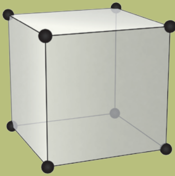
$$u \mapsto \int_f (\mathrm{tr}_f\, u) \wedge q, \quad q \in \mathcal{Q}_{r-1}^- \Lambda^{d-k}(f).$$

The spaces with constant degree r form a complex:

$$\mathcal{Q}_r^- \Lambda^0 \xrightarrow{\mathrm{d}} \mathcal{Q}_r^- \Lambda^1 \xrightarrow{\mathrm{d}} \cdots \xrightarrow{\mathrm{d}} \mathcal{Q}_r^- \Lambda^n.$$

A Q-

2 S



8

S$_1$

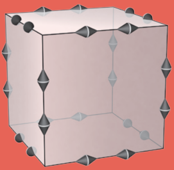
$\mathcal{S}_1\Lambda^0(\square_3)$

$8 \times \underbrace{\mathcal{P}_1\Lambda^0(\square_0)}_{1} = 8$

("S", hexahedron, 1)

S 2

3 *S*



24

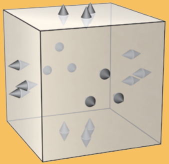
AA_1^e $\mathcal{S}_1\Lambda^1(\square_3)$

$12 \times \underbrace{\mathcal{P}_1\Lambda^0(\square_1)}_{2} = 24$

("AAE", hexahedron, 1)

3 *S*

4 S



18

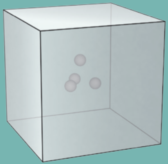
AA_1^f $\mathcal{S}_1\Lambda^2(\square_3)$

$6 \times \underbrace{\mathcal{P}_1\Lambda^0(\square_2)}_{3} = 18$

("AAF", hexahedron, 1)

S 4

5 S



4

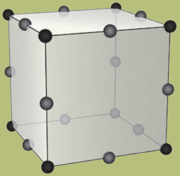
dPc_1 $\mathcal{S}_1\Lambda^3(\square_3)$

$1 \times \underbrace{\mathcal{P}_1\Lambda^0(\square_3)}_{4} = 4$

("DPC", hexahedron, 1)

5 S

6 S



20

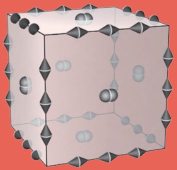
S_2 $\mathcal{S}_2\Lambda^0(\square_3)$

$$8 \times \underbrace{\mathcal{P}_2\Lambda^0(\square_0)}_{1} + 12 \times \underbrace{\mathcal{P}_0\Lambda^1(\square_1)}_{1} = 20$$

```
("S", hexahedron, 2)
```

S 9

7 S



48

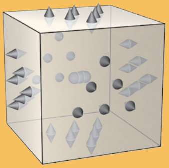
AA^{e}_{2} $\mathcal{S}_2\Lambda^1(\square_3)$

$12 \times \underbrace{\mathcal{P}_2\Lambda^0(\square_1)}_{3} + 6 \times \underbrace{\mathcal{P}_0\Lambda^1(\square_2)}_{2} = 48$

("AAE", hexahedron, 2)

7 S

8 S



39

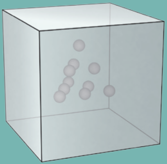
AA$^f_2$ $\mathcal{S}_2\Lambda^2(\square_3)$

$6 \times \underbrace{\mathcal{P}_2\Lambda^0(\square_2)}_{6} + 1 \times \underbrace{\mathcal{P}_0\Lambda^1(\square_3)}_{3} = 39$

`("AAF", hexahedron, 2)`

S 8

9 S



10

dPc_2

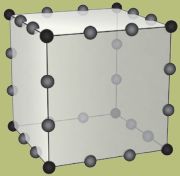
$\mathcal{S}_2\Lambda^3(\square_3)$

$1 \times \underbrace{\mathcal{P}_2\Lambda^0(\square_3)}_{10} = 10$

("DPC", hexahedron, 2)

S 6

10 S



32

S$_3$

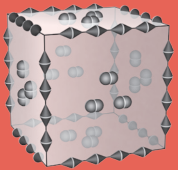
$\mathcal{S}_3\Lambda^0(\square_3)$

$$8 \times \underbrace{\mathcal{P}_3\Lambda^0(\square_0)}_{1} + 12 \times \underbrace{\mathcal{P}_1\Lambda^1(\square_1)}_{2} = 32$$

```
("S", hexahedron, 3)
```

10 S

J S



84

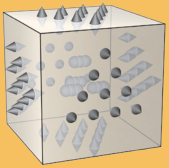
AA_3^e $\mathcal{S}_3\Lambda^1(\square_3)$

$$12 \times \underbrace{\mathcal{P}_3\Lambda^0(\square_1)}_{4} + 6 \times \underbrace{\mathcal{P}_1\Lambda^1(\square_2)}_{6} = 84$$

`("AAE", hexahedron, 3)`

S J

Q S



72

AA^f_3 $\mathcal{S}_3\Lambda^2(\square_3)$

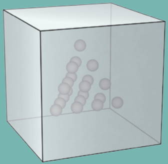
$6 \times \underbrace{\mathcal{P}_3\Lambda^0(\square_2)}_{10} + 1 \times \underbrace{\mathcal{P}_1\Lambda^1(\square_3)}_{12} = 72$

("AAF", hexahedron, 3)

S Q

K S

20



dPc$_3$ $\mathcal{S}_3\Lambda^3(\square_3)$

$1 \times \underbrace{\mathcal{P}_3\Lambda^0(\square_3)}_{20} = 20$

```
("DPC", hexahedron, 3)
```

K S

A S

$\mathcal{S}_r\Lambda^k$

The shape function space for $\mathcal{S}_r\Lambda^k$ is given by

$$\mathcal{P}_r\Lambda^k \oplus \bigoplus_{\ell\geq 1}[\kappa\mathcal{H}_{r+\ell-1,\ell}\Lambda^{k+1} \oplus \mathrm{d}\kappa\mathcal{H}_{r+\ell,\ell}\Lambda^k],$$

where $\mathcal{H}_{r,l}\Lambda^k$ consists of homogeneous polynomial k-forms of degree r which are linear and undifferentiated in at least ℓ variables.[11] Its dimension is $\dim \mathcal{S}_r\Lambda^k(\square_n) = \sum_{d\geq k} 2^{n-d}\binom{n}{d}\binom{r-d+2k}{d}\binom{d}{k}$. The degrees of freedom are

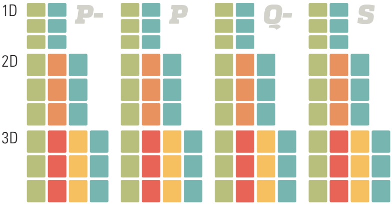
$$u \mapsto \int_f (\mathrm{tr}_f\, u) \wedge q, \quad q \in \mathcal{P}_{r-2(d-k)}\Lambda^{d-k}(f).$$

The spaces with decreasing degree r form a complex:

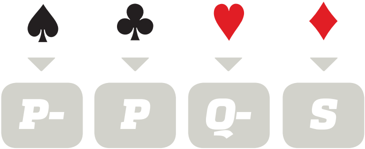
$$\mathcal{S}_r\Lambda^0 \xrightarrow{\mathrm{d}} \mathcal{S}_{r-1}\Lambda^1 \xrightarrow{\mathrm{d}} \cdots \xrightarrow{\mathrm{d}} \mathcal{S}_{r-n}\Lambda^n.$$

A S

Periodic Table of the Finite Elements



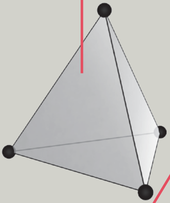
These playing cards depict the 3D elements for $r = 1, 2, 3$ of the Periodic Table of the Finite Elements. Use these cards for reference or as you would normal playing cards with the following mapping:



Legend

Element with degrees of freedom (DOFs)

Dimension of element function space



4

P$_1$

$\mathcal{P}_1^- \Lambda^0(\Delta_3)$

$4 \times \underbrace{\mathcal{P}_0 \Lambda^0(\Delta_0)}_{1} = 4$

```
("P", tetrahedron, 1)
```

Weight functions for DOFs

Element specification in FEniCS

Symbol of element

Finite element exterior calculus notation

Concept and scientific content: Douglas N. Arnold (University of Minnesota) and Anders Logg (Chalmers University of Technology). Graphic design: Mattias Schläger. The production has been supported by Simula Research Laboratory and is partially based on work supported by the U.S. National Science Foundation under grant DMS-1115291. Findings do not necessarily represent the views of Simula or of the NSF. Produced

femtable.org

FEniCS PROJECT

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- by thinking constantly about it

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