The FEniCS Project

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Workshop on Multiscale Problems and Methods
Center for Biomedical Computing, Oslo

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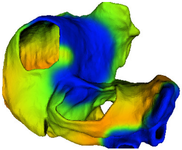
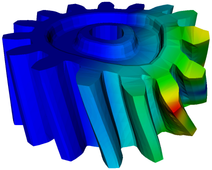




What is FEniCS?

FEniCS is an automated programming environment for differential equations

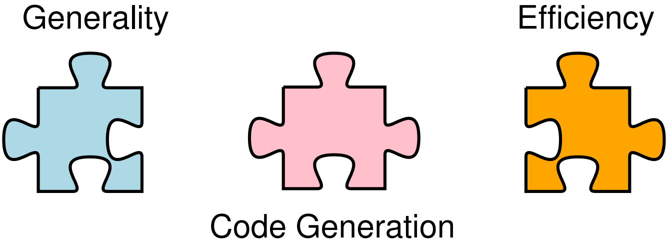
- C++/Python library
- Initiated 2003 in Chicago
- 1000–2000 monthly downloads
- Part of Debian and Ubuntu
- Licensed under the GNU LGPL

`http://www.fenicsproject.org/`

Collaborators

Simula Research Laboratory, University of Cambridge, University of Chicago, Texas Tech University, KTH Royal Institute of Technology, . . .

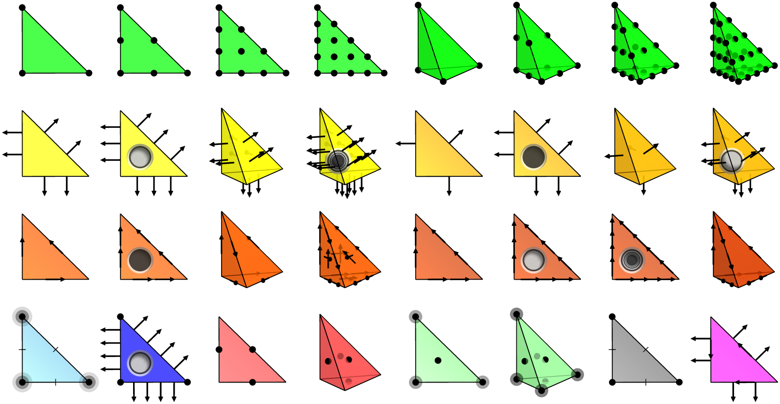
FEniCS is new technology combining generality, efficiency, simplicity and reliability



- Generality through *abstraction*
- Efficiency through *code generation*, *adaptivity*, *parallelism*
- Simplicity through *automation* and *high-level scripting*
- Reliability through *adaptive error control*

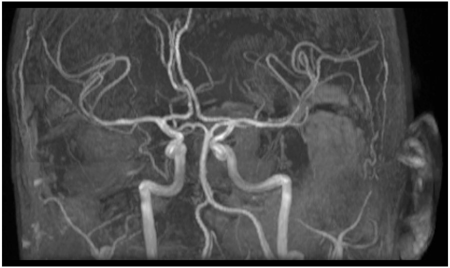
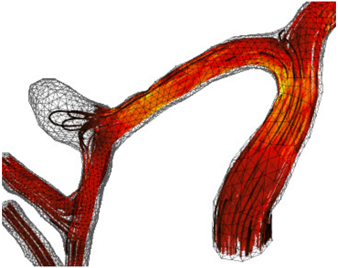
FEniCS is automated FEM

- Automated generation of basis functions
- Automated evaluation of variational forms
- Automated finite element assembly
- Automated adaptive error control



What has FEniCS been used for?

Computational hemodynamics

- Low wall shear stress may trigger aneurysm growth
- Solve the incompressible Navier–Stokes equations on patient-specific geometries

$$\dot{u} + \nabla u \cdot u - \nabla \cdot \sigma(u,p) = f$$
$$\nabla \cdot u = 0$$

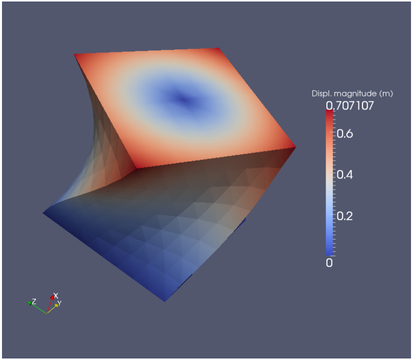
Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

Computational hemodynamics (contd.)

```
# Define Cauchy stress tensor
def sigma(v,w):
    return 2.0*mu*0.5*(grad(v) + grad(v).T)  -
w*Identity(v.cell().d)

# Define symmetric gradient
def epsilon(v):
    return  0.5*(grad(v) + grad(v).T)

# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx +
rho*inner(v, grad(u0)*(u0 - w))*dx \
   + inner(epsilon(v), sigma(U, p0))*dx \
   + inner(v, p0*n)*ds - mu*inner(grad(U).T*n, v)*ds \
   - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)

# Pressure correction
a2 = inner(grad(q), k*grad(p))*dx
L2 = inner(grad(q), k*grad(p0))*dx - q*div(u1)*dx

# Velocity correction
a3 = inner(v, u)*dx
L3 = inner(v, u1)*dx + inner(v, k*grad(p0 - p1))*dx
```

- The Navier–Stokes solver is implemented in Python/FEniCS
- FEniCS allows solver to be implemented in a minimal amount of code

Hyperelasticity




```python
class Twist(StaticHyperelasticity):

    def mesh(self):
        n = 8
        return UnitCube(n, n, n)

    def dirichlet_conditions(self):
        clamp = Expression(("0.0", "0.0", "0.0"))
        twist = Expression(("0.0",
          "y0 + (x[1]-y0)*cos(theta)
              - (x[2]-z0)*sin(theta) - x[1]",
          "z0 + (x[1]-y0)*sin(theta)
              + (x[2]-z0)*cos(theta) - x[2]"))
        twist.y0 = 0.5
        twist.z0 = 0.5
        twist.theta = pi/3
        return [clamp, twist]

    def dirichlet_boundaries(self):
        return ["x[0] == 0.0", "x[0] == 1.0"]

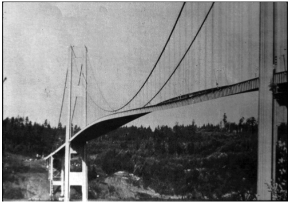
    def material_model(self):
        mu    = 3.8461
        lmbda = Expression("x[0]*5.8+(1-x[0])*5.7")

        material = StVenantKirchhoff([mu, lmbda])
        return material

    def __str__(self):
        return "A cube twisted by 60 degrees"
```

- CBC.Solve is a collection of FEniCS-based solvers developed at CBC
- CBC.Twist, CBC.Flow, CBC.Swing, CBC.Beat, . . .

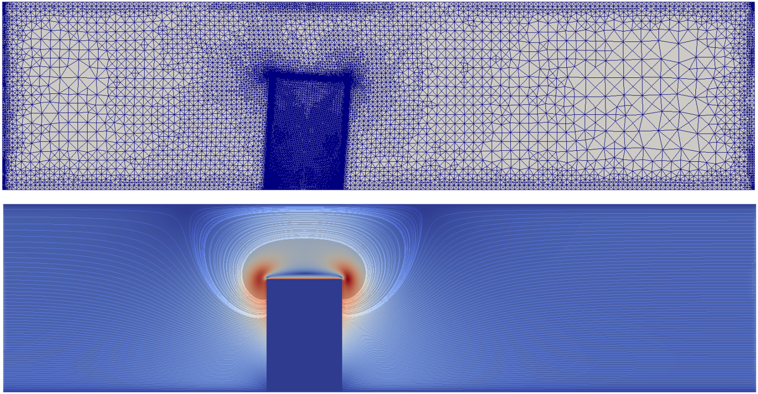
Fluid–structure interaction



- The FSI problem is a computationally very expensive coupled multiphysics problem
- The FSI problem has many important applications in engineering and biomedicine

Images courtesy of the Internet

Fluid–structure interaction (contd.)



- Fluid governed by the incompressible Navier–Stokes equations
- Structure modeled by the St. Venant–Kirchhoff model
- Adaptive refinement in space and time

Selim, Logg, Narayanan, Larson, *An Adaptive Finite Element Method for FSI* (2011)

How to use FEniCS?

Installation

Official packages for Debian and Ubuntu

Mac OS X

Drag and drop installation on Mac OS X

Binary installer for Windows

- Automated building from source for a multitude of platforms
- VirtualBox / VMWare + Ubuntu!

Hello World in FEniCS: problem formulation

Poisson's equation

$$
\begin{aligned}
-\Delta u &= f \quad \text{in } \Omega \\
u &= 0 \quad \text{on } \partial\Omega
\end{aligned}
$$

Finite element formulation

Find $u \in V$ such that

$$
\underbrace{\int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f \, v \, \mathrm{d}x}_{L(v)} \quad \forall \, v \in V
$$

Hello World in FEniCS: implementation

```
from dolfin import *

mesh = UnitSquare(32, 32)

V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]")

a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

problem = VariationalProblem(a, L, bc)
u = problem.solve()
plot(u)
```

Hello World in FEniCS: implementation

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mesh = UnitSquare(32, 32)

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v = TestFunction(V)
f = Expression("x[0]*x[1]")

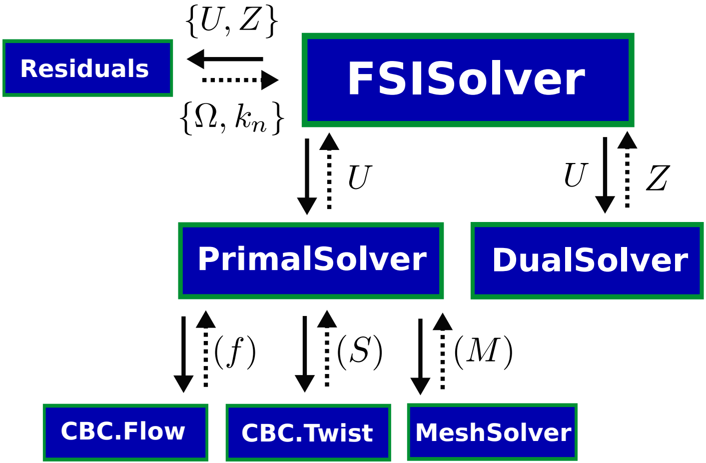
a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

A = assemble(a)
b = assemble(L)
bc.apply(A, b)

u = Function(V)
solve(A, u.vector(), b)
plot(u)
```

Implementation of advanced solvers in FEniCS



K. Selim, *An adaptive finite element solver for fluid–structure interaction problems* (2011)

Implementation of advanced solvers in FEniCS

```python
# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx +
rho*inner(v, grad(u0)*(u0 - w))*dx \
   + inner(epsilon(v), sigma(U, p0))*dx \
   + inner(v, p0*n)*ds - mu*inner(grad(U).T*n, v)*ds \
   - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)
```

```python
class StVenantKirchhoff(MaterialModel):

    def model_info(self):
        self.num_parameters = 2
        self.kinematic_measure = \
          "GreenLagrangeStrain"

    def strain_energy(self, parameters):
        E = self.E
        [mu, lmbda] = parameters
        return lmbda/2*(tr(E)**2) + mu*tr(E*E)
```

```python
class GentThomas(MaterialModel):

    def model_info(self):
        self.num_parameters = 2
        self.kinematic_measure = \
          "CauchyGreenInvariants"

    def strain_energy(self, parameters):
        I1 = self.I1
        I2 = self.I2

        [C1, C2] = parameters
        return C1*(I1 - 3) + C2*ln(I2/3)
```

```python
# Time-stepping loop
while True:

    # Fixed point iteration on FSI problem
    for iter in range(maxiter):

        # Solve fluid subproblem
        F.step(dt)

        # Transfer fluid stresses to structure
        Sigma_F = F.compute_fluid_stress(u_F0, u_F1,
                                         p_F0, p_F1,
                                         U_M0, U_M1)
        S.update_fluid_stress(Sigma_F)

        # Solve structure subproblem
        U_S1, P_S1 = S.step(dt)

        # Transfer structure displacement to fluidmesh
        M.update_structure_displacement(U_S1)

        # Solve mesh equation
        M.step(dt)

        # Transfer mesh displacement to fluid
        F.update_mesh_displacement(U_M1, dt)
```

```python
# Fluid residual contributions
R_F0 = w*inner(EZ_F - Z_F, Dt_U_F - div(Sigma_F))*dx_F
R_F1 = avg(w)*inner(EZ_F('+') - Z_F('+'),
                    jump(Sigma_F, N_F))*dS_F
R_F2 = w*inner(EZ_F - Z_F, dot(Sigma_F, N_F))*ds
R_F3 = w*inner(EY_F - Y_F,
               div(J(U_M)*dot(inv(F(U_M)), U_F)))*dx_F
```

Basic API

- `Mesh`, `MeshEntity`, `Vertex`, `Edge`, `Face`, `Facet`, `Cell`
- `FiniteElement`, `FunctionSpace`
- `TrialFunction`, `TestFunction`, `Function`
- `grad()`, `curl()`, `div()`, ...
- `Matrix`, `Vector`, `KrylovSolver`
- `assemble()`, `solve()`, `plot()`

- Python interface generated semi-automatically by SWIG
- C++ and Python interfaces almost identical

FEniCS under the hood

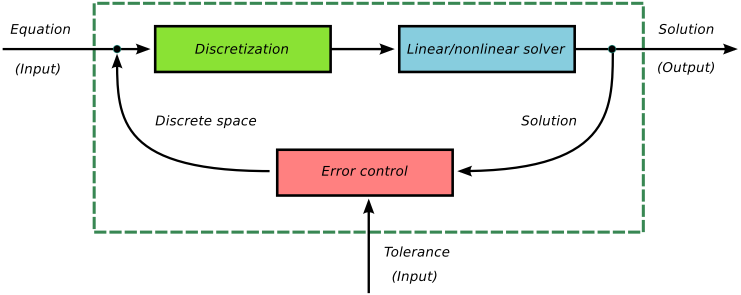
Automated scientific computing

Input

- $A(u) = f$
- $\epsilon > 0$

Output

- $u_h \approx u$
- $\|u - u_h\| \leq \epsilon$



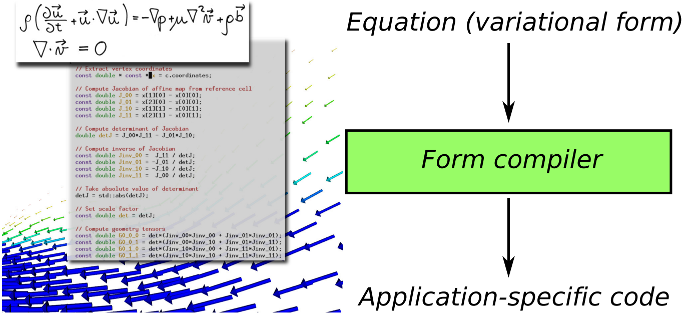
Automatic code generation

Input

Equation (variational problem)

Output

Efficient application-specific code



Code generation system

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u = Function(V)
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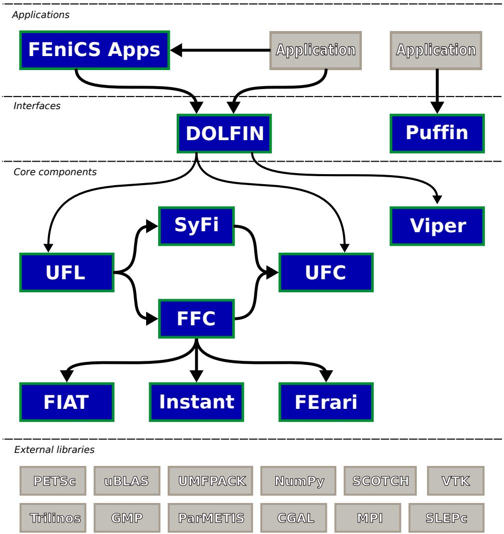
```

(Python, C++–SWIG–Python, Python–JIT–C++–GCC–SWIG–Python)

FEniCS software components



Quality assurance by continuous testing

fenics-buildbot	lucid-amd64	maverick-i386	mac-osx	linux64-exp
	9 (9) / 9	9 (9) / 9	9 (9) / 9	9 (9) / 9
ferari	Success	Success	Success	Success
fiat	Success	Success	Success	Success
ufc	Success	Success	Success	Success
instant	Success	Success	Success	Success
ufl	Success	Success	Success	Success
ffc	Success	Success	Success	building
viper	Success	Success	Success	Success
dolfin	Success	Success	Success	Success
syfi	Success	Success	Success	Success
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Closing remarks

Summary

- Automated solution of differential equations
- Simple installation
- Simple scripting in Python
- Efficiency by automated code generation
- Free/open-source (LGPL)

Upcoming events

- Release of 1.0 (2011)
- Book (2011)
- New web page (2011)
- Mini courses / seminars (2011)

`http://www.fenicsproject.org/`

`http://www.simula.no/research/acdc/`